# The Cope Rearrangement Revisited Again. Results of ab Initio Calculations beyond the CASSCF Level 

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#### Abstract

Calculations on the chair and boat Cope rearrangements have been performed at the CASPT2N level of theory, in which second-order perturbation theory is used to provide electron correlation beyond that included at the CASSCF level. Unlike previous CASSCF calculations with the 6-31G* basis set, CASPT2N calculations with 6-31G*, $6-311 \mathrm{G}^{* *}$, and $6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ all find a single stationary point of $C_{2 h}$ symmetry for the chair and $C_{2 v}$ symmetry for the boat, which corresponds to the transition state for a reaction in which bond making and bond breaking are synchronous. The CASPT2N energetics are in much better agreement with experiment than are those computed at the CASSCF/ 6-31G* level, as is the CASPT2N finding that the Cope rearrangement is concerted and does not involve the formation of a diradical intermediate.


Semiempirical and $a b$ initio electronic structure calculations have been performed in order to elucidate the mechanisms of pericyclic reactions, particularly the Cope rearrangement. ${ }^{1,2}$ The central mechanistic question in this reaction has been whether the rearrangement of 1,5 -hexadiene involves the formation of chair cyclohexane-1,4-diyl (1) as a discrete intermediate ${ }^{3 a}$ or whether the reaction is concerted and passes through a six-electron, "aromatic" transition state (2) in which bond making and bond breaking are synchronous. ${ }^{36}$


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The first calculations on the Cope rearrangement were semiempirical MINDO/2 ${ }^{5}$ and MINDO/ $3^{6}$ calculations. Both sets of calculations found only one reaction pathway, passing through 1 as an intermediate. The findings of his MINDO/3 calculations led Dewar to propose that, in general, pericyclic reactions cannot involve synchronous bond making and bond breaking.?

Subsequent AM1 calculations by Dewar initially appeared to support this contention, since only a diradicaloid intermediate

[^0](1) was found. ${ }^{8 \mathrm{a}}$ However, further investigations of the AM1 potential surface led to the claim by Dewar that a concerted, aromatic transition state (2) had also been located for the chair Cope rearrangement. ${ }^{8 b}$ Following the discovery that this purported $C_{2 h}$ transition state was not a stationary point (all first derivatives of the energy were not zero), ${ }^{9}$ it was reidentified by Dewar as a point of inflection. ${ }^{16}$ Nevertheless, AMI calculations on the boat Cope rearrangement ${ }^{8 \mathrm{~b}}$ and on the Cope rearrangements of 3,3 -dicyano-1,5-hexadiene, ${ }^{10}$ semibullvalene, ${ }^{11}$ and bullvalene ${ }^{11}$ all found two discrete $C_{2 h}$ stationary points, a minimum for 1 and a saddle point for 2.

The AM1 results have led Dewar back to the two-reaction path mechanism for the Cope rearrangement, ${ }^{16}$ which he first espoused 20 years earlier, on the basis of experiments performed in his laboratory. ${ }^{12 a}$ Kinetic studies by Dewar and Wade found that phenyl groups at either $\mathrm{C}_{2}$ or $\mathrm{C}_{3}$ of 1,5 -hexadiene accelerate the Cope rearrangement. This result led Dewar to propose the existence of two different transition states, one stabilized by phenyl groups at $\mathrm{C}_{2(5)}$ and the other by phenyl groups at $\mathrm{C}_{3(4)}$.

However, additional kinetic experiments by Dewar and Wade showed that in the Cope rearrangement of 2,4-diphenyl-1,5hexadiene, the rate accelerations found in the two monophenyl-1,5-hexadienes are multiplicative. ${ }^{12 b}$ This experimental result indicates that a single transition state is being stabilized by phenyl groups at $C_{2(5)}$ and $C_{3(4)}$. Coupled with the results of Dewar's MINDO/ 3 calculations, ${ }^{6}$ this experimental finding led Dewar to abandon the two-path mechanism, to which he has now returned on the basis of his AM1 calculations. ${ }^{16,8 b, 10,11}$

The description of the mechanism of the Cope rearrangement that has been provided by ab initio calculations has also proven to be time-dependent. Calculations with the 3-21G basis set found a $C_{2 h}$ diradical intermediate (1) when only one pair of electrons was correlated but an aromatic $C_{2 h}$ transition state (2) when correlation was provided via CI for all six of the electrons that are "active" in this rearrangement. ${ }^{13}$ The finding of an aromatic transition state for the Cope rearrangement was confirmed by complete active space (CAS)SCF/3-2lG calculations, which
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provided correlation for the six active electrons by allowing all possible occupancies of the six active orbitals. ${ }^{14}$

The aromatic transition state in this CASSCF study was found to have a bond distance, $R$, between the two allylic moieties of $2.086 \AA$. A $C_{2 h}$ diradical intermediate, $22.0 \mathrm{kcal} / \mathrm{mol}$ higher in energy, with $R=1.611 \AA$ was also located. However, we have subsequently found the latter to be an artifact, resulting from an inadvertent change in the symmetries of the six orbitals used in the active space. When the correct set of orbitals is used, this diradical geometry is only $4.8 \mathrm{kcal} / \mathrm{mol}$ above the $C_{2 h}$ transitionstate geometry, to which the diradical collapses on geometry optimization. Therefore, the only $C_{2 h}$ stationary point on the CASSCF/3-21G energy surface for the chair Cope rearrangement appears to be 2.

RHF, MP2, MP3, and MP4 calculations by Dewar and Healy all showed that the energy difference between the aromatic, $C_{2 h}$ geometry with $R=1.92 \AA$ and a higher energy diradical of $C_{2 h}$ geometry with $R=1.61 \AA$ decreases on going from 3-21G to 6-31G*. ${ }^{15}$ Subsequent CASSCF calculations with the 6-31G* basis set actually found $C_{2 h}$ stationary points for both an aromatic transition state (2) with $R=2.189 \AA$ and a diradical intermediate (1) with $R=1.641 \AA .{ }^{16}$ Diradical 1 is $1.9 \mathrm{kcal} / \mathrm{mol}$ lower in energy than 2 , but the $C_{2}$ transition state leading to 1 is only 0.6 $\mathrm{kcal} / \mathrm{mol}$ below 2.

The $C_{2 h}$ stationary points for $\mathbf{1}$ and $\mathbf{2}$ are connected by a third stationary point, which is a mountain top on the global potential energy surface and is about $3 \mathrm{kcal} / \mathrm{mol}$ above 1 . Thus, passage from the diradical to the aromatic transition-state geometry, which changes $R$ by $0.548 \AA$, results in a net energy change of only 1.9 $\mathrm{kcal} / \mathrm{mol}$ and encounters a barrier of only about $\mathrm{l} \mathrm{kcal} /$ mol more. Obviously, the CASSCF/6-31G* potential surface along a cut that preserves $C_{2 h}$ symmetry is very flat with respect to changes in $R$.

Although a six-electron, six-orbital CASSCF wave function provides correlation for the electrons that are "active" in the Cope rearrangement, it provides no correlation between these electrons and the 28 other "inactive" valence electrons. This so-called "dynamic" electron correlation is likely to be of some importance, since the CASSCF/6-31G* enthalpy of activation for passage of 1,5 -hexadiene over either the transition state leading to 1 or transition state $2^{16}$ is more than $12 \mathrm{kcal} / \mathrm{mol}$ higher than the experimental value of $33.5 \mathrm{kcal} / \mathrm{mol}$. ${ }^{4 a}$ Providing dynamic electron correlation would be expected to stabilize selectively the weak bonds in the aromatic transition state ${ }^{17 a}$ and, thus, to lower its energy with respect to both the diradicaloid intermediate and to 1,5 -hexadiene. ${ }^{17 \mathrm{~b}, \mathrm{c}}$
$A b$ initio calculations on a reaction, such as the Cope rearrangement, have an advantage over semiempirical methods in that $a b$ initio calculations can be systematically improved by increasing the amount of electron correlation that is included and by expanding the size of the basis set. In this paper we report the results of calculations on the Cope rearrangement that include dynamic electron correlation between the active and the inactive electrons, as well as correlation among the inactive electrons. We find that calculations of this type, not only with the 6-31G* basis set but also with much larger basis sets, all give a potential surface with only a single $C_{2 h}$ stationary point. This stationary point is

[^1]Table 1. Energies (hartrees) Calculated at the CASSCF/6-31G* Optimized $C_{2 h}$ Geometries for the Chair Diradical ( $R=1.641 \AA$ ) and the Chair Transition State ( $R=2.189 \AA$ ) and a Midpoint Geometry with an Interallylic Distance ( $R=1.915 \AA$ ) Halfway between These Two

|  | $R(\AA)$ |  |  |
| :--- | :---: | :---: | :---: |
| calculation | 1.641 | 1.915 | 2.189 |
| CASSCF/6-31G* | -232.9802 | -232.9762 | -232.9771 |
| CASPT2N/6-31G* | -233.7311 | -233.7301 | -233.7234 |
| CASSCF/6-311G ${ }^{* *}$ | -233.0348 | -233.0322 | -233.0345 |
| CASPT2N/6-311G** | -233.9879 | -233.9882 | -233.9818 |
| CASSCF/6-311G(2d,2p) | -233.0471 | -233.0451 | -233.0476 |
| CASPT2N/6-311G(2d,2p) | -234.0544 | -234.0585 | -234.0541 |

not a diradical intermediate (1) but the transition state (2) for a concerted Cope rearrangement, in which bond making and bond breaking are synchronous.

## Computational Methodology

Geometries were optimized with six-electron, six-orbital CASSCF calculations, using the $6-31 \mathrm{G}^{*}$ basis set. ${ }^{18}$ Geometry optimizations were performed using the Gaussian 92 suite of ab initio programs. ${ }^{19}$

For single-point calculations at these geometries, two larger basis sets, $6-311 \mathrm{G}^{* * 20}$ and $6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p}),{ }^{21}$ were also employed. The former differs from 6-31G* by including a third, independent Gaussian function for each valence orbital and polarization functions on the hydrogens. The latter includes, in addition, a second set of polarization functions, d for carbon and p for hydrogen.
Single-point calculations were performed at both the CASSCF and CASPT2N levels. CASPT2N uses second-order perturbation theory to obtain the correlation energy for all the electrons in a molecule, starting from a CASSCF reference wave function. ${ }^{22}$ CASPT2N is the multiconfigurational equivalent of MP2, ${ }^{23}$ to which it reduces for a reference wave function consisting of a single, closed-shell configuration. The CASSCF and CASPT2N calculations were performed with the MOLCAS suite of $a b$ initio programs. ${ }^{24}$

## Results

Table 1 gives both the CASSCF and CASPT2N energies with each of the three basis sets at three different $C_{2 h}$ geometries-the CASSCF/6-31 G* stationary points for 1 and $2^{16}$ and a geometry that was optimized with a value of $R=1.915 \AA$, midway between the values of $R$ at these two stationary points. Table 1 shows that with all three basis sets the CASSCF energy of the midpoint geometry is higher than that of the two endpoints. Consequently, with each basis set a barrier separates two $C_{2 h}$ stationary points on the CASSCF energy surface.
In contrast, at the CASPT2N level the midpoint geometry is computed to be lower in energy than one or both of the two endpoints with all three basis sets. This result suggests that with each basis set there is only a single $C_{2 h}$ stationary point, located somewhere near the midpoint geometry, on the CASPT2N energy surface.

[^2]Table 2. Calculated Interallylic Bond Lengths, $R(\AA)$, and Energies and Enthalpies ( $\mathrm{kcal} / \mathrm{mol}$ ), Relative to 1,5 -Hexadiene, of the Transition States for the Chair and Boat Cope Rearrangements and of Two Allyl Radicals

| calculation | chair TS |  |  | boat TS |  |  | $2 \mathrm{H}_{2} \mathrm{CCHCH}_{2}{ }^{\text {+ }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R$ | $\Delta E$ | $\Delta H$ | $R$ | $\Delta E$ | $\Delta H$ | $\Delta E$ | $\Delta H$ |
| CASSCF/3-21G ${ }^{\text {a }}$ | 2.086 | 40.7 | 37.7 | 2.316 | 47.3 | 43.6 | 54.2 |  |
| CASPT2N/3-21G | 1.890 | $29.4{ }^{6}$ |  | $2.316^{\text {c }}$ | $41.2^{\text {b }}$ |  | $61.1^{\text {b }}$ |  |
| CASSCF/6-31G*d | 2.189 | 47.7 | 45.8 | 2.615 | 52.3 | 49.0 | 54.0 | 48.1 |
| CASPT2N/6-31G* | 1.745 | $31.2^{\text {e }}$ | $30.8{ }^{\prime}$ | 2.139 | $44.0{ }^{e}$ | 42.25 | $61.6{ }^{\text {e }}$ | 55.78 |
| CASPT2N/6-311G** | 1.775 | 32.98 | $32.3{ }^{\prime}$ | 2.139 | $42.8{ }^{8}$ | 41.0 | 61.58 | 55.6 |
| CASPT2N/6-311G(2d,2p) | 1.885 | $33.1{ }^{\text {h }}$ | $32.2{ }^{\prime}$ | 2.204 | $43.3{ }^{\text {h }}$ | $41.2^{f}$ | $66.0^{h}$ | $60.1{ }^{f}$ |
| experiment |  |  | $33.5 \pm 0.5{ }^{i}$ |  |  | $44.7 \pm 2.0$ |  | $59.6 \pm 0.7^{k}$ |

${ }^{a}$ Reference $14 .{ }^{b}$ Relative to -232.2456 hartrees. ${ }^{c}$ Not reoptimized with CASPT2N. ${ }^{d}$ Reference $16 .{ }^{*}$ Relative to $\mathbf{- 2 3 3 . 7 8 1 9 \text { hartrees. } f \text { Vibrational }}$ corrections interpolated from CASSCF/6-31G* results in ref $16 .,^{g}$ Relative to -234.0384 hartrees. ${ }^{h}$ Relative to -234.1096 hartrees. ${ }^{i}$ Reference 4 a . ${ }^{j}$ Reference $27 .{ }^{k}$ Reference 28.

Because of the current unavailability of analytical energy derivatives for CASPT2N wave functions, it was not feasible to optimize fully at the CASPT2N level the geometry of the $C_{2 h}$ stationary point. However, single-point CASPT2N calculations were performed at additional, CASSCF/6-31G* optimized, $C_{2 h}$ geometries with different fixed values of $R$. These calculations allowed the value of $R$ at the $C_{2 h}$ stationary point to be located at the CASPT2N level with each basis set.

Table 2 gives the CASPT2N value of $R$ at the $C_{2 h}$ stationary point with all three basis sets. Both the large values of $R$ and the dominance of one configuration, rather than two, in the wave functions show that these $C_{2 h}$ stationary points are much better described by delocalized structure 2 than by structure 1 for cyclohexane-1,4-diyl.

A CASPT2N/6-31G* calculation was performed at a $C_{2}$ geometry in which one interallylic $\mathrm{C}-\mathrm{C}$ bond was shortened by $0.04 \AA$ from the CASPT $2 \mathrm{~N} / 6-31 \mathrm{G}^{*}$ value of $R=1.745 \AA$ for the $C_{2 h}$ stationary point, the other bond was lengthened by the same amount, and the rest of the geometry was reoptimized at the CASSCF $/ 6-31 \mathrm{G}^{*}$ level. The energy was found to go down by 0.06 mhartrees from that of the $C_{2 h}$ stationary point. This result indicates that the $C_{2 h}$ stationary point is a transition state on the CASPT2N/6-31G* energy surface.

A similar result was obtained in a CASPT2N/6-311G** calculation at a $C_{2}$ geometry in which the interallylic bond lengths were fixed at $R=1.775 \pm 0.04 \AA$. By inference, the $C_{2 h}$ stationary point with $R=1.885 \AA$ is the CASPT2N/6-311G(2d,2p) transition state for a concerted chair Cope rearrangement.

As in the case of the $C_{2 h}$ chair calculations, a single stationary point of $C_{2 v}$ symmetry for the boat Cope rearrangement was located by CASPT2N calculations with each basis set. As shown in Table 2, the CASPT2N value of $R$ for each boat stationary point was longer than that of the corresponding $C_{2 h}$ chair transition state, but considerably shorter than the CASSCF/6-31G* value of $R=2.615 \AA$ for the boat transition state. A CASPT2N/631G* calculation at a $C_{s}$ geometry verified that the $C_{2 v}$ geometry with $R=2.139 \AA$ is the transition state for a concerted boat Cope rearrangement.

The CASPT2N energies of the boat and chair transition states are given in Table 2. Also shown are the CASPT2N//CASSCF/ 6-31G* energies of two allyl radicals. ${ }^{25}$ All the CASPT2N energies are given relative to that of 1,5 -hexadiene at its CASSCF / $6.31 \mathrm{G}^{*}$ optimized geometry. The diene prefers a nonplanar $C_{2}$ to a planar $C_{2 h}$ geometry by $4.1 \mathrm{kcal} / \mathrm{mol}$ at the CASPT2N level with the $6-31 \mathrm{G}^{*}$ basis set and by $4.5 \mathrm{kcal} / \mathrm{mol}$ with $6-311 \mathrm{G}^{* *} .26$

CASPT2N calculations on 1,5-hexadiene in $C_{2}$ symmetry with the $6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ basis set proved too large to be performed.

[^3]Comparison of the 6-31G* and 6-311G** results suggests that the energy difference between the $C_{2}$ and $C_{2 h}$ conformations of 1,5 -hexadiene is unlikely to be significantly affected by further improvements in the basis set. Therefore, the CASPT2N energy of $C_{2} 1,5$-hexadiene with the $6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ basis set was estimated by subtracting from the CASPT2N energy at the optimized $C_{2 h}$ geometry the $4.5 \mathrm{kcal} / \mathrm{mol}$ difference between the CASPT2N/6-311G** energies at the optimized $C_{2 h}$ and $C_{2}$ geometries. ${ }^{26}$

In order to compare with experiment, the CASPT2N energies in Table 2 must be converted to enthalpies by correcting for the effects of differences in zero-point energies and heat capacities. The results of the CASSCF/6-31G* vibrational analyses ${ }^{16}$ were used to obtain these contributions to the enthalpy difference between two allyl radicals and 1,5 -hexadiene. The differences in zero-point energies and heat capacities between hexadiene and two allyl radicals cause the difference between their enthalpies at 500 K to be $5.9 \mathrm{kcal} / \mathrm{mol}$ smaller than the difference between their electronic energies.

Relative to 1,5 -hexadiene, the corrections for differences in zero-point energies and heat capacities to the enthalpies of the CASSCF/6-31G* $C_{2 h}$ and $C_{2 v}$ stationary points are $-0.1,-1.9$, and $-3.3 \mathrm{kcal} / \mathrm{mol}$ for, respectively, the chair diradical ( $R=$ $1.641 \AA$ ), the chair transition state ( $R=2.189 \AA$ ), and the boat transition state $(R=2.615 \AA)$ geometries. ${ }^{16}$ Since these corrections are, to a good approximation, a linear function of $R$, the corrections to the enthalpies of the CASPT2N transitionstate geometries were obtained by linear interpolations between the corrections at these three values of $R$. The resulting enthalpies are given in Table 2.

## Dicussion

The calculated enthalpy differences in Table 2 may be compared with the experimental values of $33.5 \pm 0.5 \mathrm{kcal} / \mathrm{mol}$ for the chair Cope rearrangement, ${ }^{4 \mathrm{a}} 44.7 \pm 2.0 \mathrm{kcal} / \mathrm{mol}$ for the boat, ${ }^{27}$ and $59.6 \mathrm{kcal} / \mathrm{mol}$ for cleavage of 1,5 -hexadiene to two allyl radicals. ${ }^{28}$ The CASPT2N values are obviously in far better agreement with experiment than are the CASSCF $/ 6-31 \mathrm{G}^{*}$ values, ${ }^{16}$ which are also given in Table 2.

Comparison of the CASSCF and CASPT2N results with the $6-31 \mathrm{G}^{*}$ basis set shows that the additional correlation provided by CASPT2N lowers the energy, relative to that of 1,5 -hexadiene, by $16.5 \mathrm{kcal} / \mathrm{mol}$ for the chair and by $8.3 \mathrm{kcal} / \mathrm{mol}$ for the boat transition state. At the same time the energy required to dissociate $1,5-$ hexadiene to two allyl radicals is increased by $7.6 \mathrm{kcal} / \mathrm{mol}$, so that the CASPT2N enthalpy of the boat transition state is now $13.5 \mathrm{kcal} / \mathrm{mol}$ lower than that of two allyl radicals and that of the chair $11.4 \mathrm{kcal} / \mathrm{mol}$ lower still.

CASPT2N/3-21G calculations, performed at the CASSCF/ 3-21G optimized geometries, ${ }^{14}$ give changes from the CASSCF/
(27) Goldstein, M. J.; Benzon, M. S. J. Am. Chem. Soc. 1972, 94, 7147. See also: Shea, K. J.; Phillips, R. B. J. Am. Chem. Soc. 1980, 102, 3156. (28) Roth, W. R.; Bauer, F.; Beitat, A.; Ebbrecht, T.; Wūstefeld, M. Chem. Ber. 1991, 124, 1453.

3-21G energies, similar to, but of a smaller magnitude than, those found with the 6-31G* basis set. Relative to the energy of $1,5-$ hexadiene, the CASPT2N/3-21G energies for the chair and boat transition states are, respectively, 9.6 and $6.1 \mathrm{kcal} / \mathrm{mol}$ lower than at the CASSCF/3-21G level; the dissociation energy to two allyl radicals increases by $6.9 \mathrm{kcal} / \mathrm{mol}$. As shown in Table 2, reoptimization of $R$ for the chair transition state by single-point CASPT2N/3-21G calculations at CASSCF/3-21G optimized $C_{2 h}$ geometries gives $R=1.890 \AA$ and lowers the CASPT2N/ 3-21G energy of the transition state by an additional $1.7 \mathrm{kcal} /$ mol to $29.4 \mathrm{kcal} / \mathrm{mol}$ above that of 1,5 -hexadiene.

Comparison of the 3-21G and 6-31G* results in Table 2 reveals that although the CASSCF results with the two basis sets are quite different, the CASPT2N results are rather similar. Table 2 also shows that the CASPT2N results are not much affected on going from $6.31 \mathrm{G}^{*}$ to the larger $6-311 \mathrm{G}^{* *}$ and $6.311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ basis sets. The largest effect of basis set expansion is to increase the CASPT2N $/ 6-31 \mathrm{G}^{*}$ BDE of 1,5 -hexadiene by $4.4 \mathrm{kcal} / \mathrm{mol}$ with the $6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ basis set and thus to bring it into excellent agreement with the experimental value. At the CASPT2N/6$31 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ level the calculated enthalpies of activation of both the boat and the chair Cope rearrangements are lower than the experimental values by about 1 and $3 \mathrm{kcal} / \mathrm{mol}$, respectively. ${ }^{26}$ The experimental difference between the enthalpies of the two transition states ${ }^{27}$ thus also appears to be slightly underestimated by our calculations.

The current lack of analytical derivatives for CASPT2N wave functions prevented us not only from optimizing fully at the CASPT2N level the transition-state geometries but also from performing CASPT2N vibrational analyses. Therefore, we were unable to calculate the entropies of the transition states at the CASPT2N level. However, on the basis of the CASSCF/6$31 \mathrm{G}^{*}$ results, ${ }^{16}$ the difference in calculated entropies between the two transition states is likely to be rather smaller than the experimental value of about $10 \mathrm{cal} / \mathrm{mol} \cdot \mathrm{K} .{ }^{27}$
For example, the CASSCF/6-31G* $C_{2 h}$ chair transition state is computed to have an entropy that is only $3.0 \mathrm{cal} / \mathrm{mol} \cdot \mathrm{K}$ larger than the $C_{2 h}$ chair intermediate. ${ }^{16}$ The value of $R$ for the CASPT2N boat transition state is about the same as that of the CASSCF/6-3lG* chair transition state, and the value of $R$ for the CASPT2N chair transition state is slightly larger than that of the CASSCF/6-31G* chair intermediate. Therefore, the entropy difference between the boat and chair CASPT2N transition states might be expected to be slightly less than $3 \mathrm{cal} /$ $\mathrm{mol} \cdot \mathrm{K}$. This estimate is consistent with the entropy difference of $3.8 \mathrm{cal} / \mathrm{mol} \cdot \mathrm{K}$ that is actually calculated for the CASSCF/ $3-21 \mathrm{G}$ boat ( $R=2.316 \AA$ ) and chair $(R=2.086 \AA$ ) transition states. ${ }^{14}$
As previously suggested, ${ }^{14}$ it is possible that the computed enthalpy and entropy differences between the two transition states are both smaller than those that have been measured, because the transition states actually occur not at the $C_{2 h}$ and $C_{2 v}$ geometries that minimize the electronic energies but rather at the geometries that minimize the free energies. ${ }^{29}$ Since for both $C_{2 h}$ and $C_{2 v}$ geometries, large changes in $R$ result in rather small changes in electronic energy, at 500 K changes in the entropy along $R$ can affect the location of the transition state, particularly for the boat.

For example, for the boat transition state, on going from the CASPT2N/6.311G(2d,2p) value of $R=2.244 \AA$ to the CASSCF/6-31G* value of $R=2.615 \AA$, the CASPT2N/6$311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ energy increases by $3.7 \mathrm{kcal} / \mathrm{mol}$. On the basis of the CASSCF/6-31G* entropies, ${ }^{16}$ the increase of $0.37 \AA$ in $R$ would increase the entropy by about $6 \mathrm{cal} / \mathrm{mol} \cdot \mathrm{K}$. Thus, in the free energy at 500 K this increase in entropy would offset nearly all of the increase in the enthalpy.
(29) Consequently, transition states should actually be located using variational transition-state theory: Truhlar, D. G.; Garrett, B. Acc. Chem. Res. 1980, 13, 440. Hase, W. L. Acc. Chem. Res. 1983, 16, 258.

The chair potential energy surface is also quite flat with respect to changes in $R$ along the $C_{2 h}$ cut of lowest energy. For example, on either increasing or decreasing $R$ by $0.30 \AA$ from the CASPT2N $/ 6-311 \mathrm{G}(2 \mathrm{~d}, 2 \mathrm{p})$ value of $R=1.885 \AA$ at the chair transition state, the CASPT2N/6-311G(2d,2p) energy changes by less than $3 \mathrm{kcal} / \mathrm{mol}$. Previous calculations have found that the flatness of the chair potential surface along this coordinate allows substituents to have a substantial effect on the value of $R$ in the chair transition state. ${ }^{9}$ This computational finding supports the inferences, drawn from the experiments of Gajewski and Conrad, who measured the effect of substituents on the isotope effects for bond making and bond breaking in the Cope rearrangement. ${ }^{30}$

## Conclusions

In finding that for optimized $C_{2 h}$ geometries large changes in the interallylic distance, $R$, result in only small changes in the energy, our CASPT2N calculations are similar to the previous CASSCF/6-31G* study of the Cope rearrangement. ${ }^{16}$ However, in contrast to the CASSCF/6-31G* results, CASPT2N calculations with four different basis sets find a single $C_{2 h}$ stationary point. This stationary point is the transition state (2) for a concerted chair Cope rearrangement in which bond making and bond breaking are synchronous. There is no evidence of the existence of a second stationary point, corresponding to a cyclohexane-1,4-diyl intermediate (1), which was found in the CASSCF/6-31G* calculations.

Unlike the CASSCF/6-31G* energetics, those obtained from CASPT2N calculations with $6-31 \mathrm{G}^{*}$ and with two larger basis sets are in very good agreement with experiment. With the 6-311G(2d,2p) basis set, the calculated dissociation energy of 1,5 -hexadiene to two allyl radicals is within the error limits of the experimental value. ${ }^{28}$ With the same basis set the calculated values of $\Delta H^{7}$ for the chair ${ }^{4 \mathrm{a}}$ and boat ${ }^{27}$ Cope rearrangements are, respectively, about 1 and $3 \mathrm{kcal} / \mathrm{mol}$ too low. However, at least for the boat, part of the discrepancy between the results of the calculations and the experiments may be that the $C_{2 v}$ stationary point on the free energy surface occurs at a larger value of $R$ and, hence, has both a higher enthalpy and entropy than the $C_{2 v}$ stationary point on the potential energy surface.

The dramatic improvement in the computed energetics for the chair and boat Cope rearrangements on going from CASSCF/ 6-31G* to CASPT2N/6-31G* wave functions suggests that CASPT2N may prove a generally useful method for improving upon CASSCF in calculations on pericyclic reactions. The results of CASPT2N/6-31G* calculations on the conrotatory ring opening of cyclobutene to butadiene, a reaction that has previously served as a test of various computational methodologies, ${ }^{31}$ support this conjecture. ${ }^{32}$

The excellent agreement between the CASPT2N and the experimental enthalpies of activation for the chair and boat Cope rearrangements lends support to the CASPT2N finding that both reactions are concerted and do not involve diradical intermediates. This CASPT2N result appears to be invariant to basis set expansion. Since CASPT2N provides correlation for all the electrons in 1,5 -hexadiene, not just the six electrons that are "active" in the Cope rearrangement, we believe that the CASPT2N finding of a concerted mechanism for the Cope rearrangement,

[^4]involving synchronous bond making and bond breaking, is likely to survive future calculations at even higher levels of theory. ${ }^{33}$

Experimental evidence for a concerted mechanism comes from a study by Owens and Berson. ${ }^{34}$ Their experiments show that a diradical intermediate, if formed, would have a lifetime of less than $10^{-13} \mathrm{~s}$, a lifetime shorter than that allowed for an intermediate by transition-state theory. Recently, Houk and coworkers have found that the secondary isotope effects that are calculated for a concerted chair transition state (2) are in much better agreement with the experimental values ${ }^{30}$ than the isotope effects that are calculated for formation of a diradicaloid
(33) MP4(SDTQ)/6-31G* calculations, which are based on a single reference configuration, give results that are similar to those obtained at the CASPT2N/6-31G* level. Using CASSCF/6-31G* optimized geometries, MP4(SDTQ) $/ 6-31 G^{*}$ calculations give $R=1.794 \AA$ and $E=-233.7855$ hartrees for the $C_{2 h}$ stationary point in the chair Cope rearrangement. Additional calculations show this geometry to be a transition state and to be $33.8 \mathrm{kcal} / \mathrm{mol}$ higher in energy than 1,5 -hexadiene. One way of going beyond both the CASPT2N and the single-reference MP4(SDTQ) calculations would be to implement an equivalent of MP4 with a CASSCF reference wave function. Alternatively, variational SD-CI calculations might be performed with the CASSCF wave function as reference. However, the $C_{2 h}$ CI calculations would require finding the lowest eigenvalue of a matrix representing the interactions of nearly 85000000 spin-adapted configurations.
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intermediate (1). ${ }^{35}$ Although it has been found experimentally that suitable substituents can force 1,5 -hexadiene derivatives to react by mechanisms that involve the formation of diradical intermediates, ${ }^{36,37}$ we are unaware of any experimental evidence that contradicts our computational finding of a concerted mechanism for the parent Cope rearrangement and for the rearrangement of relatively unperturbed 1,5 -hexadienes.

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